Calculus 1 — Name Key

Review – Implicit Diff. and Related Rates (1) — Block ______ Date ________________

1) Find dy/dx and the equation of the tangent line for the given function and point.

$3y+2y^2+xy=20 \qquad (3,2)$

$$3\frac{dy}{dx}+4y\frac{dy}{dx}+x\frac{dy}{dx}+y=0$$

$$\frac{dy}{dx}(3+4y+x)=-y$$

$$\frac{dy}{dx}=\frac{-y}{3+4y+x}$$

$$\frac{dy}{dx}=\frac{-y}{3+4y+x}$$

$$\frac{dy}{dx}=\frac{-2}{3+4(2)+(3)}=\frac{-2}{14}$$

$$\frac{dy}{dx}=-\frac{1}{7}$$

$$y-y_1=m(x-x_1)$$

$$y-2=-\frac{1}{7}(x-3)$$

or

$$y=-\frac{1}{7}x+\frac{17}{7}$$

2) Find dy/dx for $5xy^3-2x^3=y$.

$$5y^3+5x3y^2\frac{dy}{dx}-6x^2=\frac{dy}{dx}$$

$$5y^3-6x^2=\frac{dy}{dx}-15xy^2\frac{dy}{dx}$$

$$5y^3-6x^2=\frac{dy}{dx}(1-15xy^2)$$

$$\frac{5y^3-6x^2}{1-15xy^2}=\frac{dy}{dx}$$

or

$$\frac{dy}{dx}=\frac{6x^2-5y^3}{15xy^2-1}$$

3) Take derivative of y with respect to x for

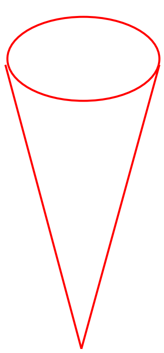
$tan(xy^2)+4x+6y=22$

$$\sec^2(xy^2)\left(x2y\frac{dy}{dx}+y^2\right)+4+6\frac{dy}{dx}=0$$

4) An inverted conical tank is being emptied at a rate of 12.6 cubic feet per second. The radius at the top of the tank is 7 feet and the height of the tank is 24 feet.

a) How fast is the height of the water changing when the depth of the water is 13ft?

b) How fast is the radius changing at the same instant?



$$\frac{dV}{dt}=-12.6\frac{ft^3}{sec}$$

Tank Dimensions

$h=24ft$

$r=7ft$

$$\frac{h}{r}=\frac{24}{7}$$

$7h=24r$

$$\frac{7}{24}h=r$$

Height of Liquid

$h=13ft$

Radius of Liquid

$$\frac{7}{24}(13)=r$$

$3.792ft=r$

$$V=\frac{1}{3}\pi r^2h$$

$$V=\frac{1}{3}\pi\left(\frac{7}{24}h\right)^2h$$

$$V=\frac{1}{3}\pi\left(\frac{7}{24}\right)^2h^3$$

$$\frac{dV}{dt}=\frac{1}{3}\pi\left(\frac{7}{24}\right)^2 3h^2\frac{dh}{dt}$$

$$-12.6=\frac{1}{3}\pi\left(\frac{7}{24}\right)^2 3(13)^2\frac{dh}{dt}$$

$$\frac{-12.6}{ans}=\frac{dh}{dt}$$

$$-.279\,{ft}/{sec}=\frac{dh}{dt}$$

$$V=\frac{1}{3}\pi r^2h$$

$$V=\frac{1}{3}\pi r^2\left(\frac{24}{7}r\right)$$

$$V=\frac{1}{3}\pi\left(\frac{24}{7}\right)r^3$$

$$\frac{dV}{dt}=\frac{1}{3}\pi\left(\frac{24}{7}\right)3r^2\frac{dr}{dt}$$

$$-12.6=\frac{1}{3}\pi\left(\frac{24}{7}\right)3(3.792)^2\frac{dr}{dt}$$

$$\frac{-12.6}{ans}=\frac{dr}{dt}$$

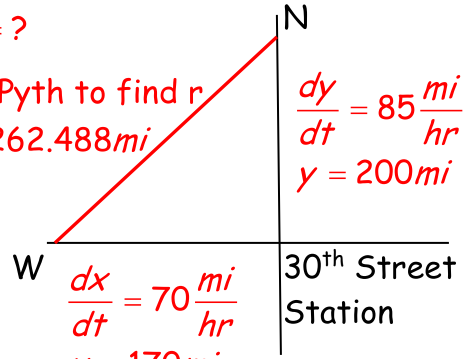
$$-.0814\,{ft}/{sec}=\frac{dr}{dt}$$

5) A train is traveling away (north) from 30th Street Station in Philadelphia at a rate of 85 miles per hour. Another train is traveling away (west) from 30th Street Station at a rate of 70 miles per hour. When the northbound train is 200 miles from Philadelphia and the westbound train is 170 miles from Philadelphia, how fast is the distance between the two trains changing?

$\frac{dr}{dt} = ?$

Use Pyth to find r

$r = 262.488mi$



$\frac{dy}{dt} = 85\frac{mi}{hr}s$

$y = 200mi$

$\frac{dx}{dt} = 70\frac{mi}{hr}$

$x = 170mi$

$x^2 + y^2 = r^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2r\frac{dr}{dt}$

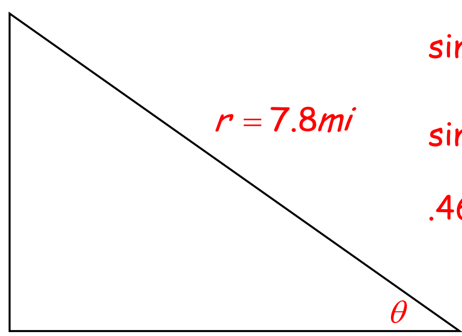
$2(170)(70) + 2(200)(85) = 2(262.488)\frac{dr}{dt}$

$110.1mph = \frac{dr}{dt}$

6) A plane flying horizontally at an altitude of 3.5 miles at a rate of 450 miles per hour is flying towards a radar station. What is the rate of change of the angle of elevation when the distance between the plane and the station is 7.8 miles?

$y = 3.5mi$

$\frac{dy}{dt} = 0$



$r = 7.8mi$

$\sin\theta = \frac{3.5}{7.8}$

$\sin^{-1}\left(\frac{3.5}{7.8}\right) = \theta$

$.465 = \theta$

Moving towards the radar station

so $\frac{dx}{dt}$ is negative

$\frac{dx}{dt} = -450mph$

Use Pyth to find x

$x = 6.971mi$

$\tan\theta = \frac{y}{x}$

Take the Derivative

$\sec^2\theta\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$

$\sec^2(.465)\frac{d\theta}{dt} = \frac{(6.971)(0) - (3.5)(-450)}{(6.971)^2}$

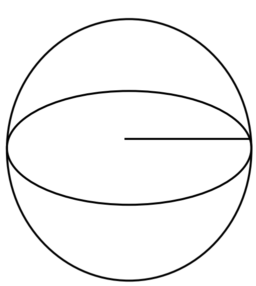
$\frac{d\theta}{dt} = ans \cdot \cos^2(.465)$

$\frac{d\theta}{dt} = 25.894\,{}^{rad}/_{hr}$

7) A spherical balloon is being deflated at a rate of 2.75 cubic centimeters per minute. At the instant the radius is 2.1 centimeters, what is the rate of change of the radius? What is the rate of change of the surface area at the same moment?

$\frac{dV}{dt} = -2.75\frac{cm^3}{\text{min}}$

$r = 2.1cm$



$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt}$

$-2.75 = \frac{4}{3}\pi 3(2.1)^2\frac{dr}{dt}$

$\frac{-2.75}{ans} = \frac{dr}{dt}$

$-.0496\,{}^{cm}/_{\text{min}} = \frac{dr}{dt}$

$SA = 4\pi r^2$

$\frac{dSA}{dt} = 4\pi 2r\frac{dr}{dt}$

$\frac{dSA}{dt} = 4\pi 2(2.1)(-.0496)$

$\frac{dSA}{dt} = -2.618\,{}^{cm^2}/_{\text{min}}$